Use the annihilator method to find the <u>form of a particular solution</u> of $y''+4y'+5y=6x^2-3e^{-2x}\sin x$. **SCORE: _____ / 6 PTS**

<u>You MUST show the usage of the annihilator. Do NOT solve for the coefficients in the particular solution.</u>

D^3 ANNIHILATES $6x^2$

$(D+2)^2+1$ ANNIHILATES $-3e^{-2x}\sin x$

$r^2+4r+5=0$

$r=-2\pm i$

1 $y_h=C_1e^{-2x}\cos x+C_2e^{-2x}\sin x$

1 1

$D^3((D+2)^2+1)(D^2+4D+5)[y]=D^3((D+2)^2+1)[6x^2-3e^{-2x}\sin x]$

$D^3((D+2)^2+1)^2[y]=0 \longrightarrow r=0,0,0,-2\pm i,-2\pm i$ $\frac{1}{2}$

$y=\underbrace{A+Bx+Cx^2}_{\frac{1}{2}}+\underbrace{De^{-2x}\cos x+Ee^{-2x}\sin x}_{y_h}+\underbrace{Fxe^{-2x}\cos x+Gxe^{-2x}\sin x}_{\frac{1}{2}}$

$\frac{1}{2}$

$y_p=A+Bx+Cx^2+Fxe^{-2x}\cos x+Gxe^{-2x}\sin x$ 1

Find the general solution of $x^2y'' + xy' + 4y = \csc(2\ln x)$. $\longrightarrow$ $y'' + \frac{1}{x}y' + \frac{4}{x^2}y$ **SCORE: _____ / 10 PTS**

$= \dfrac{\csc(2\ln x)}{x^2} = g(x)$

$r^2 + 4 = 0$

$r = \pm 2i$

(2) $y_1 = \cos(2\ln x),\ y_2 = \sin(2\ln x)$

$$W = \begin{vmatrix} \cos(2\ln x) & \sin(2\ln x) \\ -\frac{2}{x}\sin(2\ln x) & \frac{2}{x}\cos(2\ln x) \end{vmatrix} = \frac{2}{x} \quad (1)$$

$$y_p = -\cos(2\ln x)\int \frac{\frac{\csc(2\ln x)}{x^2}\sin(2\ln x)}{\frac{2}{x}}\,dx$$

$$+ \sin(2\ln x)\int \frac{\frac{\csc(2\ln x)}{x^2}\cos(2\ln x)}{\frac{2}{x}}\,dx$$

$u = \sin(2\ln x)$

$du = \frac{2}{x}\cos(2\ln x)\,dx$

$$= \text{(1)} -\cos(2\ln x)\int \frac{1}{2x}\,dx + \sin(2\ln x)\int \frac{\cos(2\ln x)}{2x\sin(2\ln x)}\,dx \quad \left(1\tfrac{1}{2}\right)$$

$$= -\cos(2\ln x)\left(\tfrac{1}{2}\ln|x|\right) \left(\tfrac{1}{2}\right) + \sin(2\ln x)\left(\tfrac{1}{4}\ln|\sin(2\ln x)|\right) \quad (1)$$

$$y = (2) -\tfrac{1}{2}\cos(2\ln x)\ln|x| + \tfrac{1}{4}\sin(2\ln x)\ln|\sin(2\ln x)|$$

$$+ c_1\cos(2\ln x) + c_2\sin(2\ln x) \quad (1)$$

BONUS QUESTION: Find a second order linear differential equation whose general solution is $y = \underbrace{Ax^2 + Bx^2 \ln x}_{y_h} + \underbrace{x^3}_{y_p}$. **SCORE: _____ / 3 PTS**

CAUCHY-EULER y_h y_p

$r = 2, 2$

$(r-2)^2 = 0$

$r^2 - 4r + 4 = 0$

① $\underline{x^2y'' - 3xy' + 4y} = b(x)$

① $\underline{x^2(6x) - 3x(3x^2) + 4(x^3)} = x^3$

① $\underline{x^2y'' - 3xy' + 4y = x^3}$

Find the general solution of $y'' - 5y' - 6y = xe^{-x}$. **SCORE: _____ / 14 PTS**

UNDETERMINED COEFFICIENTS SOLUTION

$$r^2 - 5r - 6 = 0$$
$$(r-6)(r+1) = 0$$
$$r = 6, -1$$
$$y_h = \underline{c_1 e^{6x} + c_2 e^{-x}} \quad (2)$$
$$y_p = x(Ax+B)e^{-x} = \underline{(Ax^2 + Bx)e^{-x}} \quad (2)$$
$$y_p' = (2Ax + B)e^{-x} + (-Ax^2 - Bx)e^{-x} = \underline{(-Ax^2 + (2A-B)x + B)e^{-x}} \quad (2)$$
$$y_p'' = (-2Ax + (2A-B))e^{-x} + (Ax^2 + (-2A+B)x - B))e^{-x}$$
$$y_p'' = \underline{(Ax^2 + (-4A+B)x + (2A-2B))e^{-x}} \quad (2)$$
$$-5y_p' \quad + (5A^2 + (-10A+5B)x - 5B)e^{-x}$$
$$-6y_p \quad + (-6Ax^2 - 6Bx)e^{-x}$$
$$= (2) \ \underline{(-14Ax + (2A-7B))e^{-x}} = xe^{-x}$$
$$-14A = 1 \rightarrow \underline{A = -\tfrac{1}{14}} \quad (1)$$
$$2A - 7B = 0 \rightarrow \underline{B = \tfrac{2}{7}A = -\tfrac{1}{49}} \quad (1)$$
$$y = (1)\ \underline{\left(-\tfrac{1}{14}x^2 - \tfrac{1}{49}x\right)e^{-x}} + \underline{c_1 e^{6x} + c_2 e^{-x}} \quad (1)$$

OR

VARIATION OF PARAMETERS SOLUTION

$y_1 = e^{-x} \qquad y_2 = e^{6x}$ 2

$W = \begin{vmatrix} e^{-x} & e^{6x} \\ -e^{-x} & 6e^{6x} \end{vmatrix} = 7e^{5x}$ 2

$y_p = -e^{-x}\int \frac{xe^{-x}\cdot e^{6x}}{7e^{5x}}\,dx + e^{6x}\int \frac{xe^{-x}\cdot e^{-x}}{7e^{5x}}\,dx$

2

$= -\frac{1}{7}e^{-x}\int x\,dx + \frac{1}{7}e^{6x}\int xe^{-7x}\,dx$ 2

$= -\frac{1}{7}e^{-x}\left(\frac{1}{2}x^2\right) + \frac{1}{7}e^{6x}\left(-\frac{1}{7}xe^{-7x} - \frac{1}{49}e^{-7x}\right)$

1 2

$= -\frac{1}{14}x^2e^{-x} - \frac{1}{49}xe^{-x} + Ce^{-x}$

u		$\frac{dv}{dx}$
x		e^{-7x}
1	$+$	$-\frac{1}{7}e^{-7x}$
0	$-$	$\frac{1}{49}e^{-7x}$

$y = -\frac{1}{14}x^2e^{-x} - \frac{1}{49}xe^{-x} + c_1e^{6x} + c_2e^{-x}$

2 1